

Vortex Dipole in a BEC with dipole-dipole interaction

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(Dated: January 4, 2011)

We consider single and multiply charged quantized vortex dipole in an oblate dipolar Bose Einstein condensate in the Thomas-Fermi (TF) regime. We calculate the critical velocity for the formation of a pair of vortices of opposite charge. We find that dipolar interactions decrease the critical velocity for a vortex dipole nucleation.

PACS numbers: 03.75.Nt, 03.75.Kk, 67.85.De

I. INTRODUCTION

A vortex dipole is a pair of vortices of equal and opposite circulation situated symmetrically about the origin. Under linear motion of a localized repulsive Gaussian potential, a vortex pair formation with opposite circulation is possible if the potential is moved at a velocity above a critical value [1]. In the experiments [2, 3], a laser beam focused on the center of the cloud was scanned back and forth along the axial dimension of the cigar shaped condensate. Vortices were not observed directly, but the strong heating only above a critical velocity was measured. It was shown that the measurement of significantly enhanced heating is due to energy transfer via vortex formation [4]. Recently, experimental observations of singly and multiply charged quantized vortex dipoles in a highly oblate BEC with ^{87}Rb were reported by Neely *et al.* [5]. In the experiment, vortex dipoles were created by forcing superfluid around a repulsive Gaussian obstacle using a focused blue-detuned laser beam. The beam was initially located on the left of the trap center and the harmonic potential was translated at a constant velocity until the obstacle ends up on the right of the trap center. At the same time, the height of the obstacle is linearly ramped to zero, leading to the generation of a vortex dipole that is unaffected by the presence of an obstacle or by heating due to moving the obstacle through the edges of the BEC where the local speed of sound is small. Vortex dipoles were observed to survive for many seconds in the condensate without self-annihilation. The experiment also provided evidence for the formation of multiply charged vortex dipoles. The authors in [5] noted that the theoretical predictions of critical velocity for vortex pair formation in [6] are in good agreement with the experimental results. The critical velocity is given by the minimum of the ratio of the energy to the momentum of the vortex dipole

$$v_c = \min\left(\frac{E(I)}{I}\right) \quad (1)$$

where $E(I)$ is the energy of an elementary excitation with linear momentum (or impulse) I [7]. When the object moves at a velocity above a critical value, the superfluid flow becomes unstable against the formation of quantized vortices, which give rise to a new dissipative regime [8–10]. Pairs of vortices with opposite circulation are generated at opposite sides of the object. Recently, instead of removing the trapping potential and expanding the condensate to make the vortex cores optically resolvable, Freilich et al. experimentally observed the real-time dynamics of vortex dipoles by repeatedly imaging the vortex cores [11]. Vortex tripoles have also been observed experimentally [12]. Several theoretical investigations have been reported for the generation [13–17], stability [18, 19], and stationary configurations of vortex dipoles [20, 21]. In addition, fully analytic expressions of the angular momentum and energy of a vortex dipole in a trapped two dimensional BEC were obtained [22].

The successful realization of Bose-Einstein condensation of ^{52}Cr atoms has stimulated a growing interest in the study of BEC with nonlocal dipole-dipole interactions [23–29]. A particular interest is the vortex structures in dipolar condensates [30–46]. In contrast to the isotropic character of contact interaction, long ranged and anisotropic dipole-dipole interaction has remarkable consequences for the physics of rotating dipolar gases in TF limit. It was shown that, in axially symmetric traps with the axis along the dipole orientation, the critical angular velocity, above which a vortex is energetically favorable, is decreased due to the dipolar interaction in oblate traps [42]. It was discussed that the effect of the dipole-dipole interaction is the lowered precession velocity of an off-center straight vortex line in an oblate trap [43]. Our aim in the present work is to calculate the critical velocity for vortex dipole formation in a dipolar oblate BEC in TF regime. This paper is structured as follows. Section II reviews Bose-Einstein condensates with dipole-dipole interaction. Section III investigates the critical velocity in the presence of the dipole-dipole interaction. The last section discusses the results.

II. DIPOLAR BEC

Consider a BEC of N particles with magnetic dipole moment oriented in the z direction. In the mean field theory, the order parameter $\psi(\mathbf{r})$ of the condensate is the solution of the Gross-Pitaevskii equation (GPE) [28, 29]

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_T + g|\psi(\mathbf{r})|^2 + \Phi_{dd}\right)\psi(\mathbf{r}) = \mu\psi(\mathbf{r}) \quad (2)$$

where μ is the chemical potential, $g = \frac{4\pi\hbar^2 a_s}{m}$, a_s is the s-wave scattering length, V_T is the trap potential

$$V_T = \frac{1}{2}m\omega_\perp^2 (\rho^2 + \gamma^2 z^2) \quad (3)$$

γ is the trap aspect ratio, and $\Phi_{dd}(\mathbf{r})$ is the dipolar interaction

$$\Phi_{dd}(\mathbf{r}) = \frac{3}{4\pi}g\varepsilon_{dd} \int d^3r' \frac{1 - 3\cos^2\theta}{|\mathbf{r} - \mathbf{r}'|^3} |\psi'(\mathbf{r}')|^2 \quad (4)$$

where $r - r'$ is the distance between the dipoles, θ is the angle between the direction of the dipole moment and the vector connecting the particles, and the dimensionless quantity ε_{dd} is the relative strength of the dipolar and s-wave interactions [47]. The BEC is stable as long as $-0.5 < \varepsilon_{dd} < 1$ in the TF limit [30, 47]. In the regime $\varepsilon_{dd} < 0$, the dipolar interaction is reversed by rapid rotation of the field aligning the dipoles [31, 32].

We shall use scaled harmonic oscillator units (h.o.u.) for simplicity. In this system, the units of length, time, and energy are $\sqrt{\frac{\hbar}{2m\omega_\perp}}$, $\frac{1}{2\omega_\perp}$ and $\hbar\omega_\perp$, respectively. Hence the GP equation in h.o.u reads

$$(-\nabla^2 + V'_T + g'|\psi'(\mathbf{r})|^2 + \Phi'_{dd})\psi'(\mathbf{r}) = \mu'\psi'(\mathbf{r}) \quad (5)$$

where the dimensionless interaction parameter g' is given by

$$g' = \frac{2mN}{\hbar^2} \sqrt{\frac{2m\omega_\perp}{\hbar}} g \quad (6)$$

and $\Phi'_{dd} = \Phi_{dd}(g \rightarrow g')$. The normalization of $\psi'(\mathbf{r})$ chosen here is $\int d^3r |\psi'(\mathbf{r})|^2 = 1$.

The equation (5) is an integro-differential equation since it has both integrals and derivatives of an unknown wave function. The solution of this equation was presented by Eberlein *et al.* in TF regime [47]. They showed that a parabolic density remains an exact solution for an harmonically trapped vortex-free dipolar condensate in TF limit:

$$n(\mathbf{r}) = n_0 \left(1 - \frac{\rho^2}{R^2} - \frac{z^2}{L^2}\right) \quad (7)$$

where $n_0 = \frac{\mu'}{g'}$ and R and L are the radial and axial sizes of the condensate, respectively. The condensate aspect ratio κ is defined as $\kappa = \frac{R}{L}$. In the absence of dipolar interaction, the condensate aspect ratio κ and the trap aspect ratio γ match. κ decreases with increasing ε_{dd} in an oblate trap. Hence, for $\varepsilon_{dd} > 0$ ($\varepsilon_{dd} < 0$), $\kappa < \gamma$ ($\kappa > \gamma$) [30, 31]. In h.o.u, the dipolar mean-field potential inside the condensate in the TF approximation is given by [47]

$$\Phi_{dd}(\mathbf{r}) = n_0 g' \varepsilon_{dd} \left(\frac{\rho^2}{R^2} - \frac{2z^2}{L^2} - f(\kappa) \left(1 - \frac{3}{2} \frac{\rho^2 - 2z^2}{R^2 - L^2}\right) \right) \quad (8)$$

where $f(\kappa)$ for oblate case, ($\kappa > 1$), is given by

$$f(\kappa) = \frac{2 + \kappa^2(4 - 6 \frac{\arctan \sqrt{\kappa^2 - 1}}{\sqrt{\kappa^2 - 1}})}{2(1 - \kappa^2)} \quad (9)$$

III. VORTEX DIPOLE

Vortices can be nucleated in a BEC by a localized potential moving at a velocity above a critical value [48]. Vortices with opposite circulation are generated at opposite sides of the condensate. The motion of each vortex arises from its neighbor. In the presence of weak dissipation, the two vortices slowly drift together and annihilate when the vortex separation is comparable with the vortex core size.

The critical velocity expression for a vortex dipole formation derived by Crescimanno *et al.* [6] was found in good agreement with the experimental observation [5]. We now extend their approach to include dipolar interaction. Consider a pair of single vortices with opposite charge. We suppose the vortices are located symmetrically about the trap center. Let the distance between the cores be represented by d . Substituting $\psi(\mathbf{r}) = \sqrt{n(\mathbf{r})}e^{i\phi}$ into the GP equation and equating imaginary and real terms leads to the following hydrodynamics equations:

$$\left(-\nabla^2 + (\nabla\phi)^2 + \frac{\rho^2 + \gamma^2 z^2}{4} + g'n + \Phi_{dd}\right)\sqrt{n} = \mu'\sqrt{n} \quad (10)$$

$$-\sqrt{n}\nabla^2\phi - 2\nabla\phi\cdot\nabla\sqrt{n} = 0 \quad (11)$$

The term $\nabla^2\sqrt{n}$ in the equation (10) is negligible within TF approximation. The ansatz for the phase function for a vortex dipole is [6]

$$\phi(\mathbf{r}) = l \left(\arctan\left(\frac{\sin\theta - \frac{d}{2\rho}}{\cos\theta}\right) - \arctan\left(\frac{\sin\theta + \frac{d}{2\rho}}{\cos\theta}\right) \right) \quad (12)$$

where ρ and θ are the polar coordinates, and l is vorticity. The condensate velocity is given by $v(\mathbf{r}) = \frac{\hbar}{m}\nabla\phi(\mathbf{r})$. We assume that the vortices are far enough from each other, but near enough to the trap center. The ansatz is equivalent to requiring $\frac{1}{\mu'} < d^2 < \mu'$ [6]. Using (12) we find

$$|\nabla\phi| = \left(\frac{l^2 d^2}{\rho^2 d^2 \cos^2\theta + (\rho^2 - \frac{d^2}{4})^2 + \eta} \right)^{\frac{1}{2}} \quad (13)$$

where $\eta = \frac{l^2 d^2}{\mu'}$. Excluding the vortex core regions from the domain complicates the analytic evaluation of energy and impulse precisely where the TF approximation fails. To prevent this difficulty, η is added to the denominator of (13) [6]. This regulated expression is confirmed by the observation that for vortex pair not too far from the trap center ($d^2 < \mu'$), the contribution to $\sqrt{n}(\mathbf{r})$ from the kinetic energy term $|\nabla\phi|^2$ is never larger than μ' [6].

It is reasonable to approximate the TF density in h.o.u. by

$$n(\mathbf{r}) = \frac{1}{g'} \left(\mu' - \frac{\rho^2 + \kappa^2 z^2}{4} \right) \quad (14)$$

The correction due to the term $|\nabla\phi|^2$ is at the order of d^2/R^4 , which is very small compared to R^2 in h.o.u. Let us calculate the total energy and the total impulse of the condensate. The expression of total energy of the condensate in h.o.u is given by

$$E = \int d^3\mathbf{r} \left((\nabla\sqrt{n})^2 + (\sqrt{n}\nabla\phi)^2 + \frac{(\rho^2 + \gamma^2 z^2)}{4}n + \frac{g'}{2}n^2 + \frac{\Phi'_{dd}}{2}n \right) \quad (15)$$

The total expression of impulse using the momentum of the condensate $\mathbf{P} = \frac{i\hbar}{2}[(\nabla\psi^*)\psi - \psi^*\nabla\psi]$ is

$$I = \int d^3\mathbf{r} |\mathbf{P}| = \hbar \int d^3\mathbf{r} n(\mathbf{r}) |\nabla\phi| \quad (16)$$

The critical velocity for vortex pair creation using the Landau criterion is defined as [6]

$$v_c = \frac{E_l - E_0}{I_l} \quad (17)$$

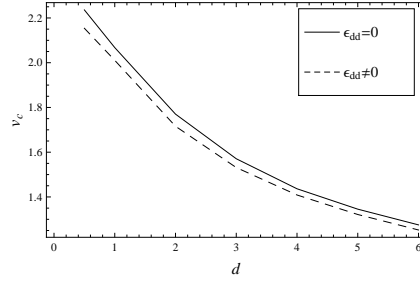


FIG. 1: The critical velocity, v_c (mm/s), for $\varepsilon_{dd} = 0$ (solid) and $\varepsilon_{dd} = 0.15$ (dashed) as a function of d in units of h.o.u. for an oblate trap with $\gamma = 5$

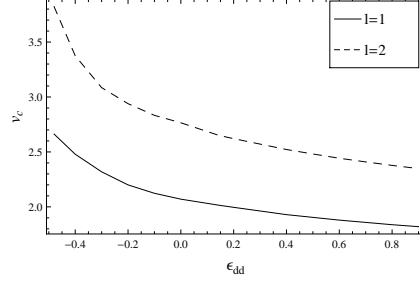


FIG. 2: The critical velocity, v_c (mm/s), for $d = 1$ in units of h.o.u. as a function of ε_{dd} for an oblate trap with $\gamma = 5$. The solid (dashed) curve is for singly (doubly) quantized vortex dipole.

Here E_l and I_l are the energy and impulse of vortex state whereas E_0 is the energy of non-vortex state. Analytical evaluation of these energy, impulse and critical velocity functions were performed by Crescimanno *et al.* in two dimensions for a nondipolar condensate [6]. In this study, we will calculate the critical velocity of a dipolar condensate in an oblate trap. As the resulting equations for v_c are complicated, it is necessary to obtain them numerically for given a_s , ε_{dd} , κ . We compare the critical velocities of dipolar and non-dipolar condensates.

IV. RESULTS

In this paper, within the TF regime we perform a numerical calculation of critical velocity for a vortex pair formation in a dipolar BEC. We examine dipolar gas containing 150000 ^{52}Cr atoms in an oblate trap with trap frequencies $\omega_{\perp} = 2\pi \times 200$ rad/s , $\omega_z = 2\pi \times 1000$ rad/s , so the trap aspect ratio is $\gamma = 5$. The magnitude of the scattering length for ^{52}Cr is $105a_B$ (a_B is the Bohr magneton). We assume that vortex separation d satisfies the condition $\frac{1}{\mu'} < d^2 < \mu'$. The chemical potential is approximately 42 in h.o.u. for $\varepsilon_{dd} = 0$ and changes very slightly with d . The dipole-dipole interaction decreases the chemical potential. Below, we perform numerical integration of

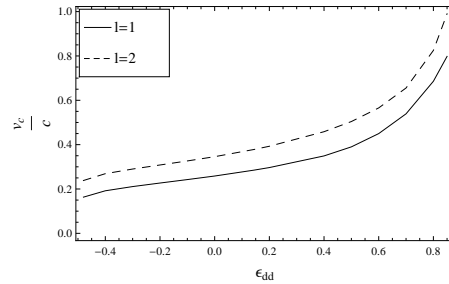


FIG. 3: The ratio of critical velocity to the speed of sound, $\frac{v_c}{c}$, for $d = 1$ in units of h.o.u. as a function of ε_{dd} for an oblate trap with $\gamma = 5$. The solid (dashed) curve is for singly (doubly) quantized vortex dipole.

energy and impulse functional (15,16) to find the critical velocity of vortex pair formation (17).

It is well known that in an oblate trap, condensate aspect ratio, κ , decreases with increasing ε_{dd} and the dipole-dipole interaction energy is positive. Minimizing the energy functional of the dipolar condensate, we find that the condensate aspect ratio increases slightly with d and the radius R is almost the same for all d . For example, κ is between 4.73 and 4.76 when d is between 0.5 and 6, respectively for $\varepsilon_{dd} = 0.15$. The kinetic energy changes significantly with vortices separation d , while the other terms in the energy expression change slightly with d . The impulse of vortex state, I , depends strongly on d and ε_{dd} since it scales as d/R^2 (13,16). It decreases with ε_{dd} , since the dipolar interaction stretches the cloud radially in an oblate trap.

The critical velocity v_c decreases with increasing separation d for a non-dipolar condensate [6]. This is because the TF density is a maximum at the trap center and reduces with the distance away from the trap center. We expect that the critical velocity decreases with d also for a dipolar BEC since parabolic form of density retains in the case of dipolar interaction. Fig-1 plots the critical velocity v_c as a function of the distance between the vortices for $\varepsilon_{dd} = 0$ (solid curve) and for $\varepsilon_{dd} = 0.15$ (dashed curve). The critical velocity is between 1.25 – 2.15 mm/s for $\varepsilon_{dd} = 0.15$, and 1.27 – 2.24 mm/s for $\varepsilon_{dd} = 0$ in the range $6 > d > 0.5$. The critical velocity difference between dipolar and non-dipolar condensates becomes smaller as d is increased. This is because κ increases with d . As can also be seen from the figure, the inclusion of dipolar interaction decreases the critical velocity for a fixed value of d . This is always true for positive values of ε_{dd} . In the case of negative values of ε_{dd} , the effect of dipole-dipole interaction is the increased critical velocity. Fig-2 shows the critical velocity as a function of ε_{dd} for fixed $d = 1$. The solid curve shows singly quantized vortex dipole while the dashed curve shows doubly quantized vortex dipole. The effect that v_c is decreased with increasing ε_{dd} in an oblate trap is the first main result of this paper. It is energetically less expensive to nucleate a vortex in an oblate dipolar Bose-Einstein condensate than in a condensate with only contact interactions. At first sight, this might seem counterintuitive since the dipole-dipole interaction energy is positive in an oblate trap. It is remarkable to note that although dipolar interaction is positive for an oblate trap, the excess dipolar energy is negative.

The nucleation of multiply charged vortex dipoles was observed for trap translation velocities well above v_c in the experiment [5]. Furthermore, it was observed that the vortices exhibit periodic orbital motion and vortex dipoles may exhibit lifetimes of many seconds, much longer than a single orbital period [5]. Fig-2 compares the critical velocity for singly and doubly quantized vortex dipole. As expected, v_c is bigger for doubly quantized vortices.

On the investigation of a vortex dipole, not only v_c , but also the ratio v_c/c is of importance. Here c is the speed of sound [27]

$$c = \sqrt{\frac{n_0}{m}g(1 + \varepsilon_{dd}(3\cos^2\alpha - 1))} \quad (18)$$

where α is the angle between directions of the wave vector and the dipoles. Suppose the direction of the phonon wave vector is perpendicular to the orientation of the dipoles ($\alpha = \pi/2$). In this case, the speed of sound becomes

$c = \sqrt{\frac{n_0}{m}g(1 - \varepsilon_{dd})}$. Remarkably, both v_c and c decrease with ε_{dd} . However, the ratio v_c/c increases with ε_{dd} . As ε_{dd} goes to one, the critical velocity approaches to the speed of sound. In Fig-3, we plot v_c/c versus ε_{dd} for singly (solid curve) and doubly quantized vortex dipoles (dashed curves) for fixed $d = 1$. The effect that v_c/c is increased with increasing ε_{dd} in an oblate trap is the second main result of this paper. The ratio v_c/c for singly quantized vortices are found to be between 0.16 – 0.31 for dipolar condensate with $\varepsilon_{dd} = 0.15$ and 0.15 – 0.28 for non-dipolar condensate in the range $6 > d > 0.5$. As expected, the ratio v_c/c increases for doubly quantized vortices.

In this paper, we have studied single and multiply quantized vortex dipoles in an oblate dipolar Bose Einstein condensate. We have shown that v_c is decreased while v_c/c is increased with increasing ε_{dd} in an oblate trap. The dynamics of vortex dipole in a dipolar BEC is worth studying. Helpful discussions with A. Kilic are gratefully acknowledged.

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